



Form follows function in a hyperboloidal cooling Tower

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Abstract

The paper proposes a computational method to study and test the relationship between shape and performance for the built environment, sustaining the use of smart geometry to support design operation. A comprehensive study of the geometry and working principles of a hyperboloidal cooling tower is presented. A case study in Marghera (Venice), built in 1938 is explored. Thanks to knowledge in descriptive geometry and mathematics, scripting procedures, digital survey, Building Information Modeling (BIM) and Computational Fluid Dynamics (CFD) analysis, the paper demonstrates how hyperboloidal cooling towers represent a reference in shape optimization. Varying the fundamental parameter of the shape, reducing the throat diameter, the simulation shows the enhanced efficiency of the tower in terms of velocity and temperature. Consequently, the proposed method may be applied to other surfaces and structures.

Keywords Hyperboloid · Geometry · Scripting · Cooling tower · CFD

Introduction

Dealing with complex surfaces, a rigorous geometric analysis seems fundamental to identify their configurative genesis, as well as to link the features of the class of the surface, to a specific case. This process emphasizes how complex the study of surfaces could be if applied to buildings. It becomes essential in such cases to consider

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the same surface as generated in different ways – changing the generatrix and its law of movement – not only for representation purposes but also in terms of quality during the phases of design, construction, and operation of the built environment. This assumption connects shape and specific performances, which measure design quality using determined parameters. The performance parameters may be related to dimensions, such as structural depth, or other metrics, such as air velocity, luminance or energy consumption.

This paper proposes a method to study and test the relationship between shape and performance for the existing built environment. The consequential parts of the proposed methodology have a foundation on descriptive geometry. In this sense, the analysis of a hyperboloidal cooling tower, built in Marghera (Venice) in 1938, represents a rare opportunity to apply and test the process here proposed (Fig. 1). Indeed, the first step is to explicitly state the geometric-mathematical importance of the hyperbolic hyperboloid configuration to increase the thermodynamic performance of the tower, compared to cylindrical or conical shapes. The second is dedicated to the specific geometry to investigate and it leads to the geometric parameters for its generation. These are used for creating an abstract 3D model within a scripting environment. The third part investigates the real built object, collecting the design documentation and the real geometry through a digital survey. Thus, a specific digital representation of the tower is created and compared to the digital survey. Given the general dimensions of the shape (height and diameters), the parametric environment can create other specific instances, assigning specific dimensions. During the BIM modelling phase, other details necessary to understand geometric features of the tower and to enable simulation are created. In the last section, a Computational Fluid Dynamics (CFD) analysis investigates all the instances created by the parametric environment, verifying enhanced performance values due to the hyperboloidal shape.

Natural Cooling Towers: Historical and Functional Framework

As machines on an architectural scale, the analysis of hyperboloidal natural cooling towers is introduced and supported by research related to three themes: the historical transformation of natural cooling towers, their technical operation, and the geometry supporting the design and construction processes.

In one of the most popular design manuals, McKelvey and Brooke offer an exposition of the transformation of cooling towers over time. According to an approach linked to the availability of technology and materials, natural cooling towers were initially wooden parallelepipeds. This framework was then replaced by steel structures, enclosed by panels made of wood or cement-based material (McKelvey and Brooke 1959). Both these structures were easily degradable and were not very efficient in terms of internal water distribution and wind resistance. The change of material is followed by a change of shape: the structures became radially symmetrical. The advent of reinforced concrete in the construction industry definitively changed cooling tower construction and formal system through continuous, cylindrical, single curved surfaces (McKelvey and Brooke 1959, 103–138). The development of the

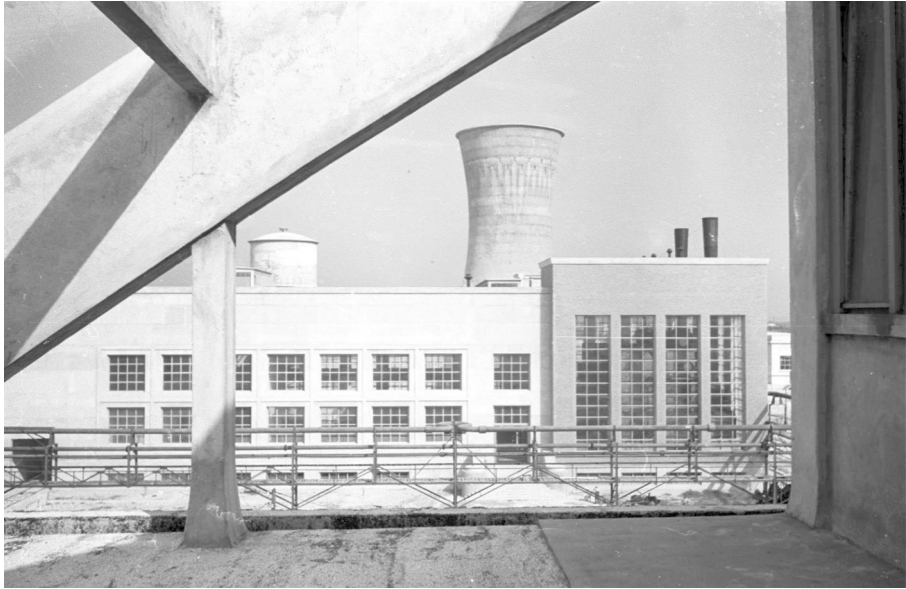


Fig. 1 Porto Marghera, Vetrocokes azotati. The tower in 1940 (Comune di Venezia, Biblioteca civica VEZ, Fondo Fotografico Giacomelli, 39,235)

shape ended with the first cooling tower construction based on the patent developed by Frederik Karel Theodoor van Iterson (1877–1957) and Gerard Kuypers, in which the authors declare “the feature that the wall [...] is not provided with any separate support or reinforcement [...] and that the wall surface for its entire height or the greatest part thereof is of double curvature in form and that the thickness of the wall varies gradually and that the wall is of a flowing shape or form in outline” (van Iterson and Kuypers 1918: 14)¹. The first hyperboloid towers built were those of the Dutch plant “Emma” between 1917 and 1918 (Mungan and Wittek 2004: 12) to meet building-structural requirements with an aerodynamic shape. The operation had to comply with high efficiency criteria by controlling the volumes of cooled water compared to the amount of material used for the construction of a “hydraulic apparatus” (van Iterson 1920). The structural behavior, and in particular the wind resistance, is the subject of further writings by the same author (van Iterson 1946)².

The cooling tower in Marghera has been recently completely refurbished and interior remodeled. Therefore, it was necessary to understand what its initial distribution apparatus was like. The reference manuals show the operation of the cooling towers in the plant water distribution system, and how this was sprayed inside the tower to produce convective motion (McKelvey and Brooke 1959). The tower has an external reinforced concrete conduit that was used to distribute the hot water to be cooled

¹ The patent was applied in the USA only by G. Kuypers (Kuypers 1920).

² The tower was built in the 1938, after 20 years of experience in hyperbolic natural cooling tower construction in Europe (Robinson 1923; Guéritte 1927; Kopp 1930; Guéritte 1932; Guéritte 1936). The work sheets mention “*Réfrigérants hyperboliques*”, probably referring to a building schema of a French patent.

from the outside to the inside, through two main access points. Inside the tower, significant voids are still visible, and they were the support of the wooden beams that were used to increase the distribution surface, making the cooling tower more efficient in terms of energy exchange.

Concerning the application of the hyperboloid to the built environment, the main reference is Vladimir G. Shukhov (1853–1939), author of the patent that first mentioned a hyperbolic lattice structure (English 2005; Beckh 2015). The geometric characteristics of the hyperbolic hyperboloid have been investigated in-depth in the existing literature (Giordano 1999; Sgrosso 2000), also regarding the Computer Aided Design (CAD) modeling of these surfaces (Saleh 1985; Prousalidou and Hanna 2007; Petkov 2009; Dragovic et al. 2014). Other authors have outlined the mathematical rules for defining the surface, describing some applications (Caliò and Marchetti 2015; Krivoschapko and Ivanov 2015; Caliò and Marchetti 2019). Considering the relationship between the surface and the hyperbola, Huylebrouck mathematically noticed how cooling towers are generated: below the throat section by a hyperbola, above the same section by an ellipse (Huylebrouck 2007). This geometric-mathematical finding has evidence in the technical literature (Gopalakrishnan et al. 1993).

Some scholars have focused on aspects of mathematical models for the study of the structural and fluid dynamic behavior of cooling towers (Nagabushana 1983; Gould and Kratzig 1999; Krivoschapko 2002; Kröger 2004). After the first efforts to identify computational models for ruled surfaces (Barbagelata 1983), many authors clarified the computational aspects through Finite Element Model (FEM) analysis (Niku 1986; Aksu 1996; Farhan et al. 2020). Likewise, the sensitivity analysis of the CFD models for the functioning of the tower was explored by others (Al-Waked and Behnia 2006; Klimanek 2013; Tanimizu and Hooman 2013), including discussing the geometric optimal parameters (El Ansary et al. 2011).

Geometric Analysis, from Generation to Tessellation

The hyperbolic hyperboloid, or one sheet hyperboloid, is an undevelopable ruled quadric surface generated by a straight line, which moves in space, constantly leaning on three straight lines, skewed two-by-two (Giordano 1999). The surface can be generated by the rotation of a hyperbola or by the rotation of a generatrix straight line around the Oz axis³ (Krivoschapko and Ivanov 2015). A tangent plane passing by any point of the surface detects two straight lines (Fig. 2), as a degenerate conic section (Sgrosso 2000).

From the study of the surface, two fundamental properties may be derived: the surface is anticlastic, having principal curvatures of opposite sign at any point, the projection of the generatrix on the horizontal plane is always tangent to the circumference at the vertex of the parabola (throat section). These properties are found in the constructive operation of hyperboloidal towers. The reinforcing bars of a concrete structure are organized along the generatrices of the form as well as the Shukhov's

³ As a condition, the second statement imposes that the generatrix and the rotation axis are skewed, in order to not create a cone.

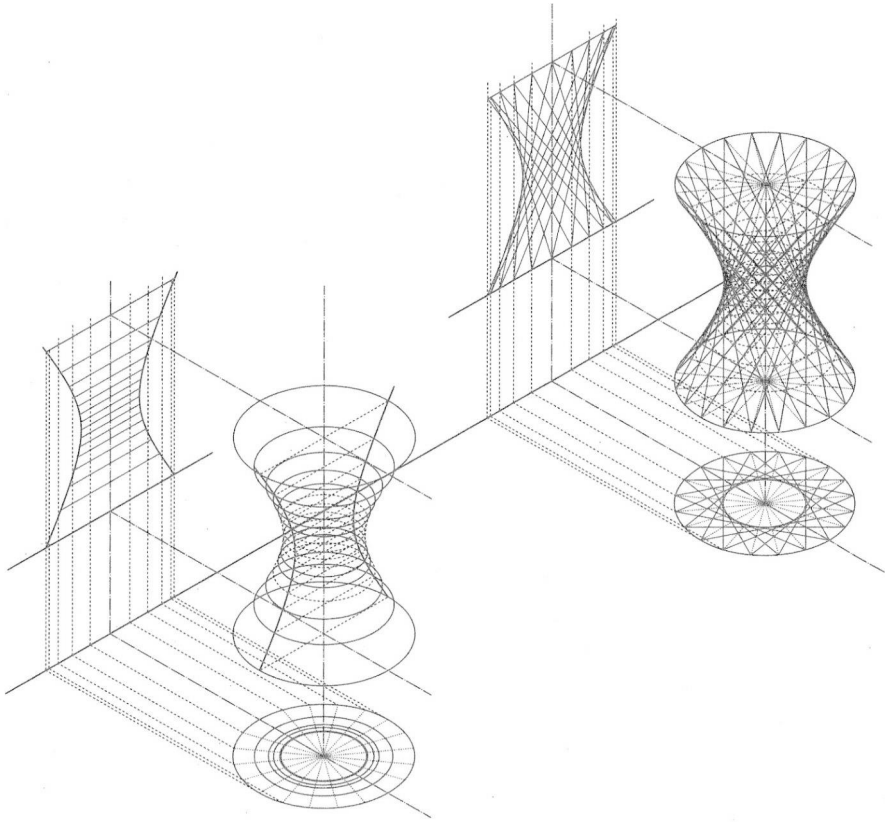


Fig. 2 Axonometric view of the same hyperbolic hyperboloid using different techniques (image by Andrea Giordano)

lattice structures. It also makes it easier the stress analysis of shell surfaces (Barbagelata 1983).

The tool used to create the surface model is a visual scripting modeling environment (Autodesk Dynamo). This environment first allows structuring a parametric framework, defining the minimum attributes which define the surface and its properties. Second, in this schema, it is possible to have two identical surfaces, by rotating the hyperbola or a generatrix around the z-axis. Ultimately, it is possible to easily change the shape adapting the surface to a specific case or creating multiple shapes with different parameter values. If the surface is created by rotating a straight line, it is defined by a starting point, at the base of the surface, and the central rotation angle HH.ph (Fig. 3). On the contrary, using a hyperbola as generatrix, it is necessary to study the correct method to describe a hyperbola through a NURBS curve (Petkov 2009; Davis 2014), by assigning one point and the vertex.

Beckh already specified the minimum geometric parameters as bottom radius (HH.Ru), top radius (HH.Ro), height ($2 \cdot \text{HH.Hh}$), number of members (HH.nn), and central rotation angle (HH.ph) (Beckh 2015). In order to create the same surface with

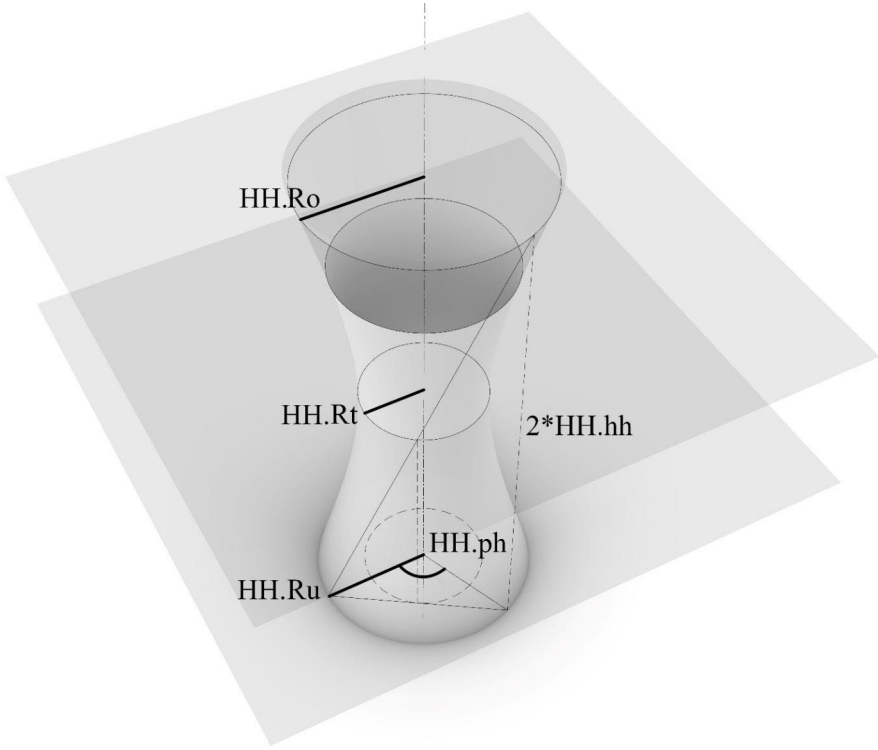


Fig. 3 Basic parameters of the hyperbolic hyperboloid (HH.Ro, HH.Rt, HH.Ru, HH.ph, HH.hh)

two methods, it is necessary to define the relationship between a straight line and the hyperbola. If the starting point of the straight line belongs to the x-axis, its coordinates (HH.Ru, 0, -HH.Hh) define a point belonging to the hyperbola. The other point is the hyperbola vertex (HH.Rt, 0, 0). From the above-mentioned condition of tangency between projection of generatrix and throat circle, the value of the x coordinate is $HH.Rt = HH.Ru(\cos(HH.ph/2))$, which is also the value of the a parameter in a hyperbola with equation:

$$\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1 \quad (1)$$

The value of c is determined by:

$$c^2 = \frac{(-HH.Hh)^2}{HH.Ru^2/a^2 - 1} \quad (2)$$

The one-sheeted circular hyperboloid with skirt radius a has cartesian equation:

$$\frac{x^2 + y^2}{a^2} - \frac{z^2}{c^2} = 1 \quad (3)$$

Its parametrical equations, for the rotation of a skewed straight line, are represented by (Beckh 2015):

$$x = x(v, \gamma) = a(\cos v \pm \gamma \cos v)$$

$$y = y(v, \gamma) = a(\cos v \pm \gamma \sin v)$$

$$z = z(\gamma) = \pm c\gamma \quad (4)$$

Given the parameters of a specific hyperbola (a,c), assigned values to γ and v ($\gamma \in \mathbb{R}$, $v \in [0, 2\pi]$), 2 points are univocally located. They represent the starting point and the endpoint of a segment of the generatrix, belonging to the horizontal planes $\pm c\gamma$.

Referring to the modeled surface, NURBS or Subdivision definition is accepted as an optimal method (Farin 1999; Antonelli et al. 2013), but simulation tools often need tessellated mesh. The problem in tessellating a hyperboloid has already been studied by Schukov, to find the best solution to place intermediate rings for lattice structures (Beckh 2015). A similar solution has been evaluated for erecting prefabricated reinforced-concrete cooling towers (McKelvey and Brooke 1959: 370). In the case of using the skewed generatrix, the prefab units could not have planar faces. Among infinite methods of tessellation, if the hyperboloid is defined by the rotation of the

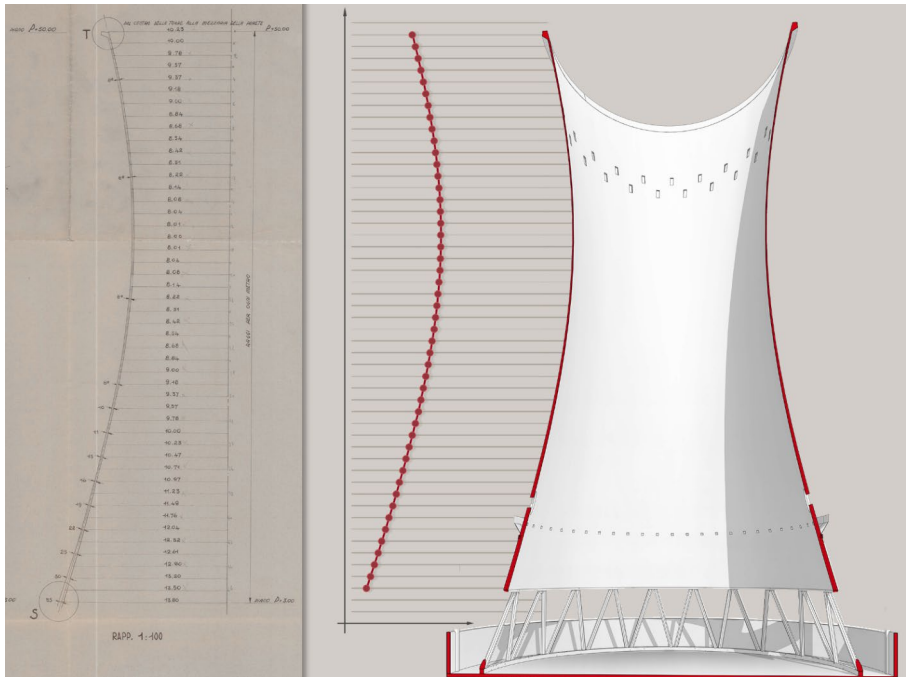


Fig. 4 On the left, designed section of the tower by assigning X, Y values (1938), on the right the as-built hyperbola in the BIM context

hyperbola, the surface is tessellated by a quad-mesh, with 4 vertices on 2 consecutive parallels. If the hyperboloid is defined by a straight line, a triangular mesh is possible by dividing on a diagonal the skewed curve obtained by two consecutive generatrices and two chords of the correspondent parallels. Another method uses triangular faces in which 2 sides are the segment of generatrix in both directions, and the other is a chord of a parallel intersected by the two generatrices.

Survey and Model the Specific Case

The case study is located in Marghera (Venice), Italy. Historical information has been gathered in two forms from private collections: design drawings and construction site annotation. The drawings are copied from a French-based design since the title blocks informed: “Refrigerant Hyperboliques – Procedure 17600 – Paris”. Refrigerant Hyperboliques was a French company, founded in 1927, among others, by two engineers Afred-Tony Guerritte and Marcel-Ernest Gérard (Association française du froid 1927). The annotation on the drawings reported 1938 as the date of design and construction.

The building and surrounding area have been digitally surveyed through Light Detection And Ranging (LiDAR) technique (Arayici and Hamilton 2005; Wang 2013), using a Leica C10 Scan Station. The representation as point cloud of the tower records the state of its conservation after structural renovation. Linking the digital survey with the generated parametric surfaces, a model is obtained that allows the representation of three surfaces, the internal, external, and median.

On design representation, the hyperbola is graphically described by a vertical radial section of the tower (Fig. 4). In the Ozx plane, it has been reported the x value – radius of any parallel – every unit meter in height, enumerated from the top to the base. Its correspondent asymmetrical hyperbola equation is:

$$\frac{x^2}{7,99^2} - \frac{z^2}{21,31^2} = 1$$

The process highlights an issue in modeling the shape from the survey. The design is defined through a curve, which generates a surface by its revolution around the z-axis. To represent the final built surfaces, starting from the curve and along its normal direction, multiple values have been assigned, representing the thickness. During the design phase, this description answers technical purposes such as volume calculation and formwork manufacture. The thickness of the tower is reported variable: at the base parallel is 35 cm, becoming 8.8 cm at 22 m, from which it remains constant. Vice versa, the digital survey represents information about the outer surfaces. Consequently, the median surface must be calculated, and it does not represent an input. The scripting environment has been changed to fulfill this requirement. The internal and external surfaces are supposed to belong to one sheet hyperboloid type.

After setting all the parameters described above, by changing the value of the angle of rotation, it is possible to superimpose graphically the ideal hyperboloidal surface with the point cloud of the real building. In its original configuration, the

tower had a total height of 50 m, it was supported by sloped columns at its base (7.10 m), with an underground fluid collection tank placed at -4.20 m. The hyperboloidal surface, not symmetrical, was 47.05 m in height (HH.hh), the base parallel inner radius was 13.56 m (HH.Ru), the top parallel radius was 10.02 m (HH.Ro), the throat section radius about 7.95 m (HH.Rt) placed at 30.60 m. The central angle of rotation of the generator is 91.7° (HH.ph).

Once set the values, the surface has been compared to the point cloud using deviance analysis. Since 58% of the points representing the internal surface are within the interval ± 3 cm, the modeling process is valid (Fig. 5).

The median surface has been built by using average values of the main parameters of the shape. Then it has been analyzed using a deviance analysis with the surface obtained from the design documentation and implemented with the formula stated above. This step makes evident the quality of the design-to-build process in 1938: the maximum distance is -3.5 cm in the upper part and 5 cm in the lower part of the surface. This result demonstrates the ability of the construction operation to build these towers without pre-fabrication techniques.

Previous steps are performed within a BIM modeling tool, to semantically enrich the 3D model and operate in a design environment. The BIM model allows for the creation of a digital twin of the original design and the actual structure (4D model). First, the surface was completed at the top and bottom, as a closed surface, represented by a unique wall entity (IfcWall) (ISO 2018). Thus, it is possible to associate to it the designed rebars along the generatrices of the hyperboloid, with alternatively positive and negative values of the angle HH.ph (Fig. 6). The model is then completed with other elements, which are design solutions particularly related to geometry or performance of the tower. For example, the tower included two openings for the plant water inlet. They have an elliptical profile to maximize the space between two consecutive circular and rectilinear reinforcements. Additionally, the tower presented small holes on the top to reduce precipitation and recover some water by condensation. They have been modeled as an object based on the wall (IfcOpening).

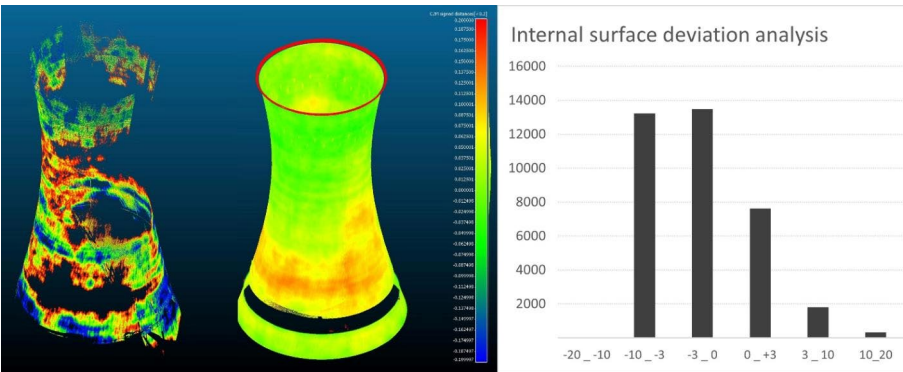


Fig. 5 Deviance analysis between the internal surface of the model and the point cloud

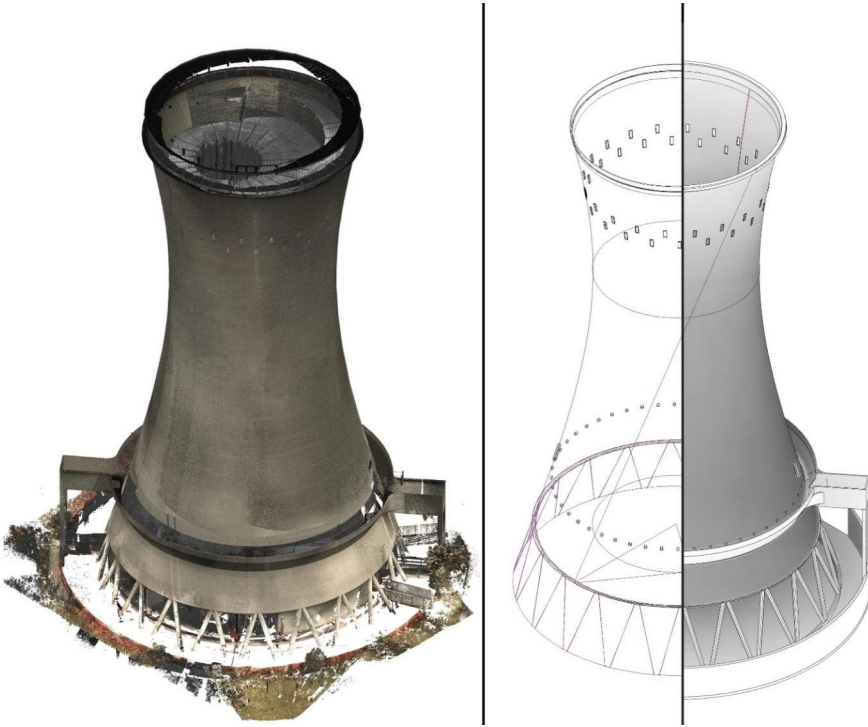


Fig. 6 On the left, the representation of the tower as point cloud. On the right, the analytical structural model and the BIM model presenting the geometric entities from the scripting environment

The top and bottom of the surface were later mapped with two circular structural objects (IfcBeam) that have their own reinforcements. The slanted columns (IfcColumn) at the ground floor, designed to support the tower and let the cooling fluid in, have a hexagonal section. Their analytical representations are congruent with the beam described above. The model is finally completed with the slabs and the retaining walls of the base water tank (Fig. 6). The median surface of the tower, fitting for FEM and CFD model, is defined as an instance of IfcWall with appropriate representation through NURBS surface.

Shape-Based Performance Analysis

Coming to the analysis of the performance, the main scope in a cooling tower is to cool the transfer fluid. To test the shape's impact – relative to other hyperboloids, cylinder and cone – on the final performance, CFD analyses have been carried out. They have been paired with the use of interoperable formats which allowed a smooth exchange of information and a transfer to the simulation environment without geometric and information loss. The parametric study allows assigning different values of the central rotation angle (HH.ph), which correspond to export different geometric models: considering the smallest angle value the tower is cylindrical, instead consid-

ering an angle greater than 120° the shape tends to become a cone. To start the simulation, the components, and related data concern: the surrounding environment, the assumptions of simplifications to describe the system and finally the varied geometry of the tower.

In natural cooling towers, the lifting force is due to the difference in density between the outgoing air, which is warmer and humid, and the ambient air: the operation of the tower is therefore strongly influenced by the environmental conditions, which have therefore been carefully analyzed to define a reliable scenario and then enable the shape changes.

To set equal boundary conditions for the different simulations, it was necessary to determine the input parameters through a reverse engineering process, considering: the performance data of similar structures of the first half of the 20th century, the variables and processes which incur in cooling by evaporation and convection. These are distinctly linked to the values of the physical quantities of the water in the circuit and to the conditions of the environment in which the structure is located.

Without exhaustive data on the full operation of the original tower, the inlet temperature and the variation between the inlet and outlet temperature are taken from the scientific literature, as cited in the first section. They were chosen considering the construction date, the material used, the entrance way of the water, the presence or absence of packaging and possibly its material. Then, evaluating the geometric and the environmental data of the place where they were installed, the different flow rates per covered area were compared, obtaining that of the tower in Marghera and then determining the reference flow rate and the inlet water temperature.

To establish the other boundary conditions to be set in the simulation, the following practice dynamics were considered.

The tower is sized referring to the bulb temperature exceeded only in 5% of the hours of the period between June and September, referring to Europe. Therefore, it is often able to ensure the desired cooling, that is, in all months of the year and in 95% of the hottest hours of the summer. Therefore, considering the environmental data available in the Marghera area, the surrounding urban conformation without chief obstacles, two environmental scenarios have been set, in summer and winter, the first one shows the optimum geometric condition in the worst condition, the second in the best ones. After defining the scenarios, to start the simulation it was necessary to create a volume equivalent to the action of hot water, continuously entering at the base of the tower. The power value was determined with previously defined parameters, while the 3D geometry was created parametrically in the BIM software to be associated to the shape hyperbolic hyperboloid. Eight configurations have been established, with steps of variation of the angle $HH.ph$ of 10° , to have appreciable variations in results and exclude the shape's degeneration at the external values.

The CFD software used to perform the analysis is VENTO AEC, which allows for importing the IFC file generated before, with both semantic and geometric data. An outdoor simulation was performed, using the values obtained from the literature to define the initial values and the boundary condition of the fluid domain, in this case, air. The algorithm performs an average of 150 iterations to reach the convergence value in each simulation.

The results were compared in terms of velocity, pressure, and temperature, as well as the expression of the draft, inversely linked to temperature (Table 1). It is interesting to underline that due to the complexity that characterizes the natural draft cooling process, the analysis of the draft, mathematically expressed through the volumetric thermal expansion coefficient, is more understandable and verifiable thanks to a graphic expression obtainable only through a fluid dynamic simulation (Fig. 7). The fluid velocity of the existing tower against the configuration with angle HH.ph as 60° highlights better performance in the designed configuration.

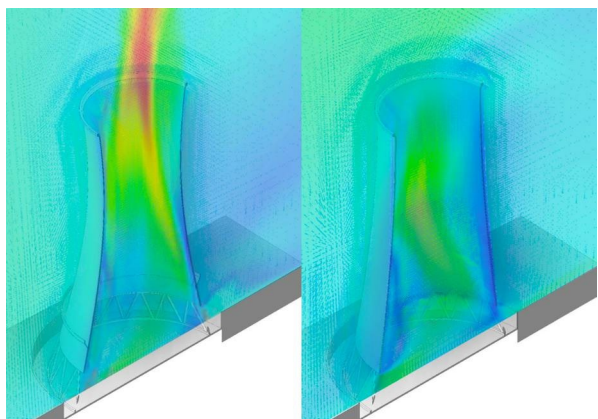
Conclusions

This paper is part of a larger research project to support the use of smart geometry in the built environment. Some existing structures encompass technical knowledge that used geometry to support design operation. The paper proposes a method to investigate in-depth, the relationship between shape and performance. The process aims

Table 1 Results of the computational fluid dynamic analysis. Variation of the geometry, variable: central rotation angle (HH.ph)

HH.ph	VELOCITY m/s		TEMPERATURE °C		PRESSION Pa		DENSITY Kg/mc	
	min	max	min	max	min	max	min	max
91.7°	0.0025	8.56	6.83	77.86	101,297	101,344	1.010	1.26
75°	0.0033	8.55	6.40	74.68	101,297	101,345	1.020	1.26
60°	0.0040	7.84	6.30	73.17	101,299	101,341	1.010	1.27
45°	0.0040	7.71	3.08	71.87	101,298	101,348	1.023	1.29
30°	0.0034	7.69	6.73	71.96	101,299	101,349	1.023	1.26
15°	0.0040	7.71	5.86	72.17	101,299	101,349	1.022	1.27
105°	0.0040	8.51	6.47	77.95	101,296	101,343	1.010	1.26
120°	0.0040	8.64	6.83	79.60	101,296	101,343	1.000	1.26

Fig. 7 CFD Analysis of the cooling tower working operation by change the rotation angle, representing velocity. On the left the as-built configuration, on the right the configuration with HH.ph 60°



to create comprehensive knowledge, based on geometry, for the restoration of these structures or as an example for new design development.

The method involved multidisciplinary interconnected steps, taken from reverse engineering processes, such as literature and geometrical analysis, digital survey, scripting and BIM modeling, simulations. The steps operate under standard geometrical and mathematical constraints, such as defining maximum tolerance for survey, BIM exchange requirements, simulation procedures and algorithms. Moreover, smooth information exchanges involving geometric and performance parameters are needed.

To check the methodology proposed, the hyperboloid for natural cooling tower, representing a cooling machines in an architectural scale was selected. The process demonstrated how the hyperboloidal shape increases the draft of the air and evenly distributes the water reaching its full cooling function. Thanks to a parametric BIM environment and CFD analysis, the research tested different geometric models, corresponding to specific values of the central rotation angle (HH.ph). Increasing the angle, and consequently reducing the throat diameter, the simulation showed the enhanced efficiency of the tower in terms of velocity and temperature. The results support the general validity of the methodology.

Additionally, the process creates a BIM model, functional for the maintenance of the building, and for historical research about architecture, infrastructure and technology. The model can also work as the base for the communication of industrial heritage.

The development of each step presented specific challenges and original contributions, such as the script to create a double representation of the hyperbolic hyperboloid, or the results from CFD analysis, presented as a comparison of multiple simulations. Future steps of this research include performing scripting techniques to highlight, in a hyperbolic hyperboloid, the contemporaneous optimization of both structural behavior and natural drafts.

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